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FEEDBACK CONTROL SYSTEM

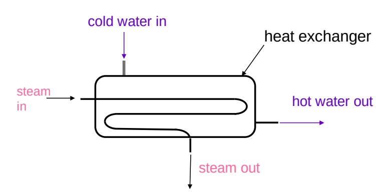
- Process control is methods to force process parameters to have specific values.
- Objective to maintain the value of some quantity at some desired level regardless of influences

Process Types

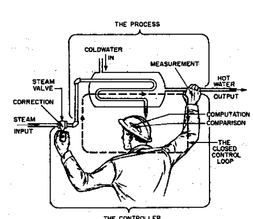
- **Self-regulating Processes**
  - These are uncontrolled processes. The process variables are not regulated. Example!
- **Manual Controlled Processes**
  - These proceses are controlled by human Example!
- **Automatic Controlled Processes**
  - These process are controlled by automatic controller. There are 2 types:
    - feed forward control system. Example!
    - feedback control system (closed-loop control system). Example!
  - We concern with the analysis and design of closed-loop control system.

Self-regulating Processes

- The process outputs are not regulated, its value will easily change.
- following is an example of self regulating process

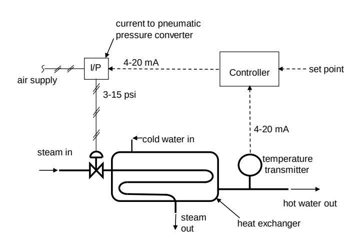


Manual controlled process



Heat exchange process when under Manual control. The dot line represent the closed loop of the controller and the process

Automatic controlled process

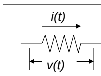
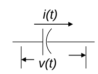
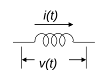


Heat exchange process when under Automatic control.

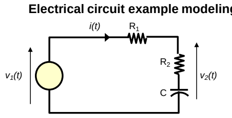
MODELS OF PHYSICAL SYSTEM

- Mathematical model of a system is defined as a set of equation used to represent physical system.
- It should be understood that no mathematical model of physical system is exact, although we may increase the accuracy by increasing the complexity of the equations
- In this module we only concern with LTI system, whose equation can be solved using Laplace transform and can be represented by a transfer function.

models of electrical elements

Component	Diff. equ.	Laplace transform
	$v(t) = Ri(t)$	$V(s) = RI(s)$
	$v(t) = \frac{1}{C}\int_0^t i(t)dt + v(0)$	$V(s) = \frac{1}{sc}I(s)$
	$v(t) = L\frac{di}{dt}$	$V(s) = LsI(s)$

Electrical circuit example modeling



- In this circuit we consider v_1 to be the input and v_2 to be the output.
- here we have to write a set equations whose solution will yield $v_2(t)$ as function of $v_1(t)$ or $V_2(s)$ as a function of $V_1(s)$

$$R_1 i(t) + R_2 i(t) + \frac{1}{C}\int_0^t i(\tau)d\tau = v_1(t)$$

$$R_2 i(t) + \frac{1}{C}\int_0^t i(\tau)d\tau = v_2(t)$$

$$R_1 I(s) + R_2 I(s) + \frac{1}{sC}I(s) = V_1(s)$$

$$R_2 I(s) + \frac{1}{sC}I(s) = V_2(s)$$

From the first Laplace transformed equation

$$R_1 I(s) + R_2 I(s) + \frac{1}{sC}I(s) = V_1(s)$$

we solve for $I(s)$

$$I(s) = \frac{V_1(s)}{R_1 + R_2 + (1/sC)}$$

Substituting $I(s)$ to the second equation

$$R_2 I(s) + \frac{1}{sC}I(s) = V_2(s)$$

we get

$$V_2(s) = \frac{R_2 + (1/sC)}{R_1 + R_2 + (1/sC)}V_1(s)$$

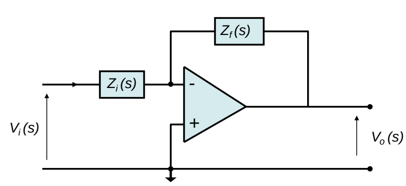
The transfer function of this system is

$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2 sC + 1}{(R_1 + R_2)sC + 1)}$$

Electrical circuit example modeling

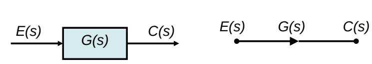
- thus the circuit can be modeled by:
  - two differential equation
  - two equation in the LAPLACE transform variable, or
  - a transfer function
- another model using state space will be discussed next time

Op-amp example modeling

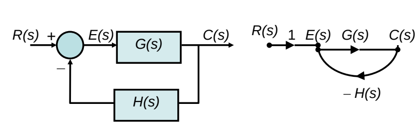


$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_f(s)}{Z_i(s)}$$

Block diagram and signal flow graph

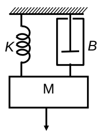


$C(s)= G(s)E(s)$



$$C(s) = \frac{G(s)}{1 + G(s)H(s)}R(s)$$

Mechanical Translational System modeling

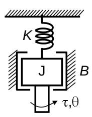


$$f(t) = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

$$C(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

$$F(s) = Ms^2X(s) + BsX(s) + KX(s) = (Ms^2 + Bs + K)X(s)$$

Mechanical Rotational System modeling



$$\frac{\theta(s)}{T(s)} = \frac{1}{Js^2 + Bs + K}$$

STATE VARIABLE MODELING

- Purpose: to develop presentation which preserves the input output relationship, but which is expressed in n first order equation
- Advantage: in addition to the input-output characteristic, the internal characteristic of the system is represented
- Computer aided analysis and design of state models are performed more easily
- We feedback more information (internal/state variable) about the plant
- Design procedure that result in the best control system are almost all based on state variable models.

STATE VARIABLE MODELING

Let us start with an example

$$M\frac{d^2y(t)}{dt^2} = f(t) - B\frac{dy(t)}{dt} - Ky(t)$$

$$G(s) = \frac{Y(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

For second order system we define two state variables $x_1(t)$ and $x_2(t)$ as

$x_1(t) = y(t)$

$$x_2(t) = \frac{dx_1(t)}{dt} = \dot{x}_1(t)$$

Then we may write

$$\frac{d^2y(t)}{dt^2} = \frac{dx_2(t)}{dt} = \dot{x}_2(t)$$

$$= -\frac{B}{M}x_2(t) - \frac{K}{M}x_1(t) + \frac{1}{M}f(t)$$

Written in specific format, we have

$$\dot{x}_1 = x_2$$

$$\dot{x}_2(t) = -\frac{K}{M}x_1(t) - \frac{B}{M}x_2(t) + \frac{1}{M}f(t)$$

$$y(t) = x_1(t)$$

In matrix notation, we have

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{K}{M} & -\frac{B}{M}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{M}\end{bmatrix}f$$

$$y(t) = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

A second order D.E. has been modified into two first order D.E's. we used two state variables x_1 and x_2. For one n order D.E there will be n first order D.E's having n state variables.

STATE VARIABLE MODELING

- Definition: The state of a system of any time t_0 is the amount of information at t_0 that together with all inputs for $t \geq t_0$, uniquely determines the behavior of the system for all $t \geq t_0$
- The standard form of the state equation is
  dx(t)/dt = Ax(t) + Bu(t)
  y(t) = Cx(t) + Du(t)

  where
  - **x(t)** = state vector
  - **A** = (n×n) system matrix
  - **B** = (n×r) input matrix
  - **u(t)** = input vektor = (r×1) vector composed of the system input function
  - **y(t)** = output vektor = (p×1) vector composed of the defined output
  - **C** = (p×n) output matrix
  - **D** = (p×r) matrix to represent direct coupling between input and output

STATE VARIABLE MODELING

- Recall that standard form of the state equation is
  dx(t)/dt = Ax(t) + Bu(t)
  y(t) = Cx(t) + Du(t)
- The first equation, called the state equation, is a 1st order D.E and **x(t)** is the solution of the equation.
- The second one is the output equation. Given **x(t)** and **u(t) y(t)** can be found.
- Usually matrix **D** is zero. Nonzero **D** indicates that there are some path coupled input and output.
- On the first equ only the first derivatives of the state var may appear on the left side of equation and no derivatives on the right side
- No derivatives may appear on the output equation.
- The standard format of the state equation valid for multiple input and output system.

STATE VARIABLE MODELING

Example 3.1 Consider a D.E as follows:

$$\ddot{y}_1 + k_1\dot{y}_1 + k_2y_1 = u_1 + k_3u_2$$
$$\dot{y}_2 + k_4y_2 + k_5\dot{y}_1 = k_0u_1 \qquad (1)$$

where u_1 and u_2 are inputs and y_1 and y_2 are outputs. Let us define the states:

$$x_1 = y_1 \quad x_2 = \dot{y}_1 = \dot{x}_1$$
$$x_3 = y_2 \qquad (2)$$

from (1) and (2) we write :

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -k_2x_1 - k_1x_2 + u_1 + k_3u_2$$
$$\dot{x}_3 = -k_5x_2 + k_4x_3 + k_0u_1$$

with the outputs equation

$$y_1 = x_1$$
$$y_2 = x_3$$

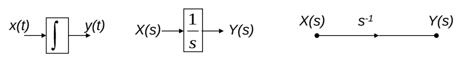
this equation equations may be written in matrix form

$$\dot{\mathbf{x}} = \begin{bmatrix}0 & 1 & 0\\ -k_2 & -k_1 & 0\\ 0 & -k_5 & -k_4\end{bmatrix}\mathbf{x} + \begin{bmatrix}0 & 0\\ 1 & k_3\\ k_6 & 0\end{bmatrix}\mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\mathbf{x}$$

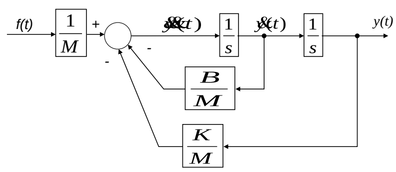
SIMULATION DIAGRAM

We have presented the way of finding state model from differential Equation. We will present a method of finding state model form transfer function. The method is based on simulation diagram. It is a block diagram or flow graph consisted of gain, summing junction and integrator only.



Symbols of integrator

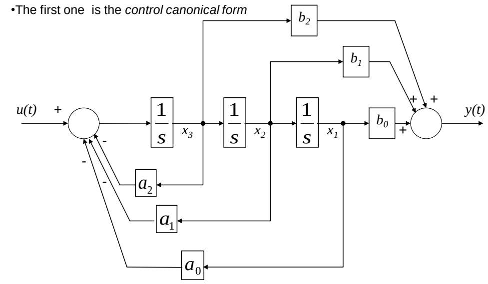
We will construct simulation diagram of $\ddot{y}(t) = -\left(\frac{B}{M}\right)\dot{y}(t) - \left(\frac{K}{M}\right)y(t) + \left(\frac{1}{M}\right)f(t)$



SIMULATION DIAGRAM

The transfer function of third order system is: $G(s) = \dfrac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$

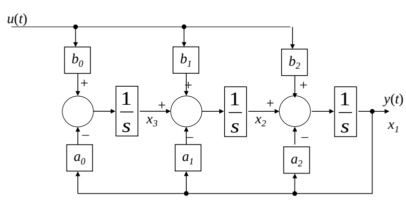
- There are two common form of simulation diagram
- The first one is the *control canonical form*



SIMULATION DIAGRAM

The second one is the *observer canonical form* :

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_2 s^2 + a_1 s + a_0}$$



The diagrams can be easily expanded to higher order system

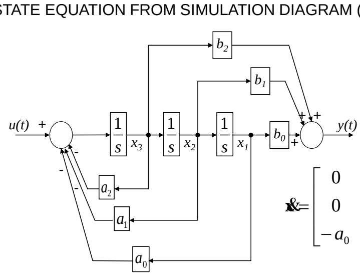
SIMULATION DIAGRAM

Once simulation diagram of transfer function is constructed, a state model of the system is easily obtained. The procedure has two step

1. Assign a state variable to the output of integrator
2. Write an equation for the input of each integrator and an equation for each system output . These equation are written as function of integrator outputs and the system inputs

This procedure yields the following state equation

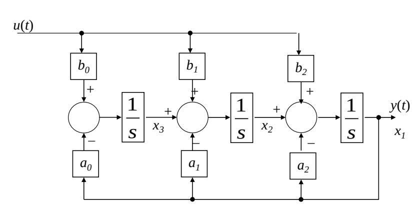
STATE EQUATION FROM SIMULATION DIAGRAM (control canonical form)



$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix}\mathbf{x}$$

STATE EQUATION FROM SIMULATION DIAGRAM (observer canonical form)



$$\dot{\mathbf{x}} = \begin{bmatrix} -a_2 & 1 & 0 \\ -a_1 & 0 & 1 \\ -a_0 & 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} b_2 \\ b_1 \\ b_0 \end{bmatrix}\mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{x}$$

Example

Consider the mechanical system with the following transfer function

$$G(s) = \frac{Y(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K} = \frac{\frac{1}{M}}{s^2 + \frac{B}{M}s + \frac{K}{M}}$$

The state model of control canonical form is

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{B}{M} \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f(t)$$

$$y(t) = \begin{bmatrix} \frac{1}{M} & 0 \end{bmatrix}\mathbf{x}(t)$$

The state model of observer canonical form is

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -\frac{K}{M} & 1 \\ -\frac{B}{M} & 0 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} f(t)$$

$$y(t) = \begin{bmatrix} & 0 \end{bmatrix}\mathbf{x}(t)$$